TA Session 2: Agglomeration

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1 Introduction

In this TA session, we will cover the following:

- One micro-foundation of increasing returns at the city level.
  - Through this example, we will also see the workhorse production or demand framework in modern macroeconomics.
- One recent effort of estimating the strength of agglomeration in Ahlfeldt et al. (2015).
  - Through this example, we will see a version of the model that we saw last week in a famous paper and how it is used for estimation.

2 Increasing Returns from Sharing

In this section, we will go over one possible micro-foundation of aggregate increasing returns from sharing discusses in Duranton and Puga (2004).

2.1 Production Overview

- The production side consists of two types of firms:
  1. A continuum of intermediate good producers that uses labor to produce varieties of intermediate goods;
  2. A final good producer that combines intermediate goods to produce a final good for consumption.
- Intermediate good producer operates a linear technology featuring fixed costs that leads to increasing returns to scale at the firm level. The intermediate good markets are monopolistically competitive and the labor market is perfectly competitive.

- Final good producer operates a constant elasticity of substitution (CES) technology. The final good market is perfectly competitive.
- Some notation for aggregate quantities and prices:
  - Y: Output of final good.
  - n: Number of intermediate goods.
  - L: Total labor.
  - W: Wage rate.
- We want to show that this setup delivers us the results that a proportional increase in L (an equilibrium object) implies a more than proportional increase in Y (also an equilibrium object). Some notes:
  - We do not need to fully solve the equilibrium, but simply need to derive the relationship between these two equilibrium objects.
  - The aggregate increasing returns to scale mainly relies on the property that each intermediate good would be in "fixed" supply regardless of the aggregates. Because of this, more L leads to more varieties and consequently more output Y. The production technology ensures that the increase is more than one for one.

2.2 Final Good Producer

- Its production function is given by

$$Y = \left(\int_0^n y(i)^{\frac{\sigma-1}{\sigma}} \mathrm{d}i \right)^{\frac{\sigma}{\sigma-1}}$$

  where $\sigma > 1$ represents the elasticity of substitution.
  - This is what economists call a constant elasticity of substitution (CES) production function. It is also a constant returns to scales (CRS) production function.
- The firm's cost minimization problem is given by

$$\min_{y(i)} \int_0^n p(i)y(i) \quad \text{s.t.} \quad \left(\int_0^n y(i)^{\frac{\sigma-1}{\sigma}} \mathrm{d}i \right)^{\frac{\sigma}{\sigma-1}} = Y.$$

  - The Lagrangian is given by

$$\mathcal{L} = -\int_0^n p(i)y(i) + \lambda \left[\left(\int_0^n y(i)^{\frac{\sigma-1}{\sigma}} \mathrm{d}i \right)^{\frac{\sigma}{\sigma-1}} - Y \right]$$

The KKT conditions are

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial y(i)} &= -p(i) + \lambda \left(\int_0^n y(i)^{\frac{\sigma-1}{\sigma}} \mathrm{d}i \right)^{\frac{\sigma}{\sigma-1}-1} y(i)^{-\frac{1}{\sigma}} = 0 \qquad \forall i \in [0,n] \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= \left(\int_0^n y(i)^{\frac{\sigma-1}{\sigma}} \mathrm{d}i \right)^{\frac{\sigma}{\sigma-1}} - Y = 0
\end{aligned}$$

– We can derive an expression of λ from the first equation:

$$\begin{aligned}
& p(i) = \lambda \left(\int_0^n y(i)^{\frac{\sigma-1}{\sigma}} \mathrm{d}i \right)^{\frac{\sigma}{\sigma-1}-1} y(i)^{-\frac{1}{\sigma}} \\
\Rightarrow \quad & p(i)^{-(\sigma-1)} = \lambda^{-(\sigma-1)} \left(\int_0^n y(i)^{\frac{\sigma-1}{\sigma}} \mathrm{d}i \right)^{-1} y(i)^{\frac{\sigma-1}{\sigma}} \\
\Rightarrow \quad & \int_0^n p(i)^{-(\sigma-1)} \mathrm{d}i = \lambda^{-(\sigma-1)} \\
\Rightarrow \quad & \lambda = \left(\int_0^n p(i)^{-(\sigma-1)} \mathrm{d}i \right)^{-\frac{1}{\sigma-1}}
\end{aligned}$$

This is what we usually use as the "ideal price index" for CES demand systems. We will use the notation $P \equiv \lambda$. Note that in our case it has the interpretation of the marginal value of an additional unit of output.

– Now we can express the demand of intermediate good i given Y and $p(i)$ as

$$\begin{aligned}
& p(i) = PY^{\frac{\sigma-1}{\sigma}(\frac{\sigma}{\sigma-1}-1)} y(i)^{-\frac{1}{\sigma}} \\
\Rightarrow \quad & y(i) = Y \left(\frac{p(i)}{P} \right)^{-\sigma}.
\end{aligned}$$

– The firm's cost function is then

$$C(Y) = \int_0^n p(i) Y \left(\frac{p(i)}{P} \right)^{-\sigma} \mathrm{d}i = Y P^{\sigma} P^{-(\sigma-1)} = PY.$$

- The firm's profit maximization problem is then

$$\max_Y P_F Y - PY.$$

We know that in equilibrium the price of final good P_F has to equal to P and the firm earns 0 profit.

- Additional, To see why this functional form is called CES, let us calculate the elasticity of substitution from the demand function derived above:

$$\frac{\partial y(i)}{\partial p(i)} \Big/ \frac{y(i)}{p(i)} = \left(-\sigma \frac{Y}{P}\left(\frac{p(i)}{P}\right)^{-\sigma-1}\right) \Big/ \left(\frac{Y}{p(i)}\left(\frac{p(i)}{P}\right)^{-\sigma}\right) = -\sigma.$$

2.3 Intermediate Good Producers

- Its production function is given by

$$y(i) = \beta l(i) - \alpha,$$

  where $l(i)$ denotes its labor input, β denotes its marginal productivity of labor, and α denotes the fixed cost of production.

- Each intermediate good producer faces monopolistic competition in the product market and perfect competition in labor market. Its profit maximization problem is given by

$$\max_{p(i),l(i)} \; p(i)(\beta l(i) - \alpha) - W l(i)$$
$$\text{s.t. } \beta l(i) - \alpha = Y\left(\frac{p(i)}{P}\right)^{-\sigma}$$

  – The choice of $p(i)$ does not affect P because each intermediate producer is small (of measure 0).

  – The firm's maximization problem can be reformulated as

$$\max_{p(i)} \; p(i) Y \left(\frac{p(i)}{P}\right)^{-\sigma} - W \frac{Y\left(\frac{p(i)}{P}\right)^{-\sigma} + \alpha}{\beta}.$$

  The first order condition is

$$(1-\sigma)p(i)^{-\sigma} + \sigma \frac{W}{\beta} p(i)^{-\sigma-1} = 0,$$

  which gives us

$$p(i) = \frac{\sigma}{\sigma - 1}\frac{W}{\beta}.$$

  Therefore, the firm will charge a constant markup $\frac{\sigma}{\sigma-1}$ over its marginal cost $\frac{W}{\beta}$.

- Because there is an infinite number of potential entrants, in equilibrium it has to be the case that intermediate good producers earn 0 profit, which implies

$$\begin{aligned}
&\frac{\sigma}{\sigma-1}\frac{W}{\beta}(\beta l(i)-\alpha)-Wl(i)=0 \\
\Rightarrow \quad & l(i)=\sigma\frac{\alpha}{\beta}, \quad y(i)=(\sigma-1)\alpha.
\end{aligned}$$

Note how this result is independent of aggregate prices and quantities.

2.4 Aggregate Output and Labor

- We now want to determine the relationship that Y, n, L needs to satisfy in equilibrium.
- As we shown above, in equilibrium, each intermediate good producer hire the same number of workers $l(i)=\sigma\frac{\alpha}{\beta}$, which implies that n should satisfy

$$n=\frac{L}{l(i)}=\frac{\beta}{\sigma\alpha}L.$$

- Y should then satisfy

$$\begin{aligned}
Y &= \left(\int_0^n y(i)^{\frac{\sigma-1}{\sigma}}\mathrm{d}i\right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\int_0^{\frac{\beta}{\sigma\alpha}L}((\sigma-1)\alpha)^{\frac{\sigma-1}{\sigma}}\mathrm{d}i\right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\frac{\beta}{\sigma\alpha}L((\sigma-1)\alpha)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\frac{\beta}{\sigma\alpha}\right)^{\frac{\sigma}{\sigma-1}}(\sigma-1)\alpha L^{1+\frac{1}{\sigma-1}}
\end{aligned}$$

  We can see that Y increases more than proportionally as L increases proportionally because $1+\frac{1}{\sigma-1}>1$.

- "An increase in final production by virtual of sharing a wide variety of intermediate suppliers requires a less-than-proportional increase in primary factors."

3 The Berlin Wall Paper (Ahlfeldt et al., 2015)

3.1 Model Setup

Overview

The model is essentially what we see last week with some modifications.

- The city consists of a set of discrete locations.
- Floor space produced with land can be used for residential or commercial purposes.
- Firms producing a freely traded final good choose locations to produce.
- Workers choose (1) whether to move to the city, (2) locations to reside and work, and (3) consumption of final good and housing service. Commuting cost is exogenous.
- In comparison to the model in Bordeu (2023):
  - Commuting is simpler.
  - Agglomeration and land market are more complicated for estimation purpose.

Locations

- $i \in \mathcal{S} = \{1, \ldots, S\}$: S discrete locations.
- K_i: Total land.
- L_{Mi}, L_{Ri}: Total floor space for production and residential purposes.
- H_{Mi}, H_{Ri}: Total count of workers for production and residential purposes.
- τ_{ij}: Travel time between locations i and j.
- A_j: Productivity in location j with functional form

$$A_j = a_j \Upsilon_j^{\lambda}, \qquad \Upsilon_j \equiv \left[\sum_{s=1}^{S} e^{-\delta \tau_{is}} \left(\frac{H_{Ms}}{K_s} \right) \right],$$

  where

  - a_j is productivity fundamentals.
  - Υ_j is spillovers.
  - δ is the rate of decay of spillovers.
  - λ captures the relative importance of spillovers.

- B_i: Amenities in location i with functional form

$$B_i = b_i \Omega_i^{\eta}, \qquad \Omega_i \equiv \left[\sum_{s=1}^{S} e^{-\rho \tau_{is}} \left(\frac{H_{Rs}}{K_s} \right) \right],$$

where

- b_j is amenity fundamentals.
- Ω_j is spillovers.
- η is the rate of decay of spillovers.
- ρ captures the relative importance of spillovers.

Firms

- Perfectly competitive firm in location j produces a freely traded final (numerie) good with production technology

$$y_j = A_j \left(H_{Mj}\right)^{\alpha} \left(L_{Mj}\right)^{1-\alpha}, \qquad 0 < \alpha < 1.$$

- Prices that the firm in location j faces:
  - 1: Output price (numerie).
  - w_j: Wage.
  - q_i: Commercial rent for floor space.
- Firms choose a location for production, employment, and commercial floor space to maximize profits taking as given goods and factor prices, productivity and the locations of other firms/workers.

Workers

- Workers make the following choices:
  1. Whether to move to the city before observing their idiosyncratic preferences for locations within the city.
  2. Where to live and commute as well as expenditure on consumption and housing after observing their idiosyncratic preferences for locations within the city.
- Worker ω's utility function when residing in i, working in j, consuming good c_{ij} and housing l_{ij} is given by

$$U_{ij\omega} = \frac{B_i}{d_{ij}} \left(\frac{c_{ij}}{\beta}\right)^{\beta} \left(\frac{\ell_{ij}}{1-\beta}\right)^{1-\beta} z_{ij\omega}, \qquad 0 < \beta < 1,$$

where

- d_{ij} is commuting cost given by

$$d_{ij} = e^{\kappa \tau_{ij}}.$$

- $z_{ij\omega}$ is idiosyncratic preference shock drawn from a Fréchet distribution:

$$F(z_{ij\omega}) = e^{-T_i E_j z_{ij\omega}^{-\epsilon}}, \qquad T_i, E_j > 0,\ \epsilon > 1.$$

- Prices that workers living in location i and working in location j faces:
  - 1: Consumption price.
  - w_j: Wage.
  - Q_i: Residential rent for floor space.
- Worker's utility of living outside of the city is $\overline{U}$.
- This preference structure allows us to derive a set of commute flow equations that help characterize the equilibrium:

$$\pi_{ij} = \frac{T_i E_j \left(d_{ij} Q_i^{1-\beta}\right)^{-\epsilon} (B_i w_j)^{\epsilon}}{\sum_{r=1}^{S} \sum_{s=1}^{S} T_r E_s \left(d_{rs} Q_r^{1-\beta}\right)^{-\epsilon} (B_r w_s)^{\epsilon}} \equiv \frac{\Phi_{ij}}{\Phi}.$$

Land Market (Crucial Difference)

- Land market equilibrium requires no-arbitrage between the commercial and residential use of floor space after the tax equivalent of land use regulations.
  - The share of floor space used commercially is

$$\theta_i \begin{cases} = 1 & \text{if} \quad q_i > \xi_i Q_i \\ \in [0,1] & \text{if} \quad q_i = \xi_i Q_i \\ = 0 & \text{if} \quad q_i < \xi_i Q_i \end{cases},$$

    where $\xi_i \geq 1$ captures one plus the tax equivalent of land use regulations that restrict commercial land use relative to residential land use.
  - The authors assume that the observed price of floor space in the data is the maximum of the commercial and residential price of floor space $\mathbb{Q}_i = \max\{q_i, Q_i\}$ with the following relationship:

$$\begin{aligned} \mathbb{Q}_i &= q_i, \quad q_i > \xi_i Q_i, \quad \theta_i = 1 \\ \mathbb{Q}_i &= q_i, \quad q_i = \xi_i Q_i, \quad \theta_i \in [0,1] \\ \mathbb{Q}_i &= Q_i, \quad q_i < \xi_i Q_i, \quad \theta_i = 0. \end{aligned}$$

- Floor space is supplied by a competitive construction sector that uses land K and capital M as inputs with technology:

$$L_i = \underbrace{M_i^{\mu}}_{\equiv \varphi_i} K_i^{1-\mu}.$$

- The corresponding dual cost function for floor space is

$$\mathbb{Q}_i = \underbrace{\mu^{-\mu}(1-\mu)^{-(1-\mu)}\mathbb{P}^{\mu}}_{\equiv \chi} \mathbb{R}_i^{1-\mu}$$

where $\mathbb{P}$ is the common price for capital and $\mathbb{R}_i$ is the price for land.

- The two equations above summarize the relationships between the quantities and prices of floor space and land.

3.2 Empirics

Model Inversion

- "We show that there is a unique mapping from the observed variables to unobserved location characteristics [conditional on parameters]. These unobserved location characteristics include production and residential fundamentals and several other unobserved variables."

- **Proposition 2** (A Key Result)

  1. Given known values for the parameters $\{\alpha, \beta, \mu, \varepsilon, \kappa\}$ and the observed data $\{\mathbb{Q}, \mathbf{H}_M, \mathbf{H}_R, \mathbf{K}, \tau\}$, there exist unique vectors of the unobserved location characteristics $\{\tilde{\mathbf{A}}^*, \tilde{\mathbf{B}}^*, \tilde{\varphi}^*\}$ that are consistent with the data being an equilibrium of the model.

  2. Given known values for the parameters $\{\alpha, \beta, \mu, \varepsilon, \kappa, \lambda, \delta, \eta, \rho\}$ and the observed data $\{\mathbb{Q}, \mathbf{H}_M, \mathbf{H}_R, \mathbf{K}, \tau\}$, there exist unique vectors of the unobserved location characteristics $\{\tilde{\mathbf{a}}^*, \tilde{\mathbf{b}}^*, \tilde{\varphi}^*\}$ that are consistent with the data being an equilibrium of the model.

The Berlin Wall

- Four channels through which division or reunification affects the distribution of economic activity within West Berlin:

  1. A loss of employment opportunities in East Berlin;

2. A loss of commuters from East Berlin;
3. A loss of production externalities from East Berlin;
4. A loss of residential externalities from East Berlin.

- Expected effects of division:
  - Reduces overall population;
  - Reduces floor prices, workplace employment, and residence employment in parts of West Berlin closer to employment and residential concentrations in East Berlin relative to those elsewhere in West Berlin.

Reduced-Form Evidence

- Estimate difference in difference specification for division and reunification separately (for areas in West Berlin):

$$\Delta \ln O_i = \alpha + \sum_{k=1}^{K} \mathbb{I}_{ik}\beta_k + \ln M_i \gamma + u_i.$$

- They find negative coefficients associated with the division and positive coefficients associated with the reunification. Moreover, coefficients are larger for areas closer to East Berlin, in line with the model prediction.

Estimating $\tilde{A}_i, \tilde{B}_i, \tilde{\varphi}_i$ Recursively

- The authors first apply Proposition 2 Item (1) to estimate $\tilde{A}_i, \tilde{B}_i, \tilde{\varphi}_i$ without dealing with agglomeration parameters $\lambda, \delta, \eta, \rho$.
- The model has a recursive structure that allows the authors to find the following unknowns in sequence (see Section 3.6 in Ahlfeldt et al. (2015) for more details):

  1. Known: π_{ij}.
     Relationship: Commuting gravity equation.
     Unknown: $\nu = \varepsilon\kappa$ semi-elasticity of commuting flows with respect to travel times.
  2. Known: $H_{Mj}, H_{Ri}, \tau_{ij}$.
     Relationship: Worker commuting probabilities.
     Unknown: $\tilde{w}_j$.
  3. Known: $\tilde{w}_j, \mathbb{Q}_i$.
     Relationship: Firm's cost function.
     Unknown: $\tilde{A}_j$.

4. Known: $\tilde{w}_j, \mathbb{Q}_i, \theta_i$.
   Relationship: Worker's utility maximization and population mobility.
   Unknown: $\tilde{B}_j$.

- The authors then use the division and reunification events as counterfactuals to show that the exogenous location characteristics is unable to explain the observed impact of division and reunification.

Estimating $\tilde{a}_i, \tilde{b}_i, \tilde{\varphi}_i$ and Parameters with Generalized Method of Moments (GMM)

- While the previous exercise demonstrates the importance of agglomeration effects, it cannot distinguish externalities from adjusted fundamentals in $\tilde{A}_i, \tilde{B}_i$. The authors managed to separate the two components using the exogenous variation induced by Berlin's division and reunification.

- Key moment condition: The changes in adjusted production and residential fundamentals are uncorrelated with the exogenous change in the surrounding concentration of economic activity induced by Berlin's division and reunification.
  - Basically, the change in production and residential fundamentals cannot be correlated with location's distance to the Berlin Wall.
  - Then the systematic change in the gradient of economic activity in West Berline relative to the pre-war CBD following division will be indicative of model mechanisms.

- Findings:
  - Recall that the functional forms are:

$$
\begin{aligned}
A_j &= a_j \Upsilon_j^{\lambda}, \qquad \Upsilon_j \equiv \left[\sum_{s=1}^{S} e^{-\delta \tau_{is}} \left(\frac{H_{Ms}}{K_s} \right) \right], \\
B_i &= b_i \Omega_i^{\eta}, \qquad \Omega_i \equiv \left[\sum_{s=1}^{S} e^{-\rho \tau_{is}} \left(\frac{H_{Rs}}{K_s} \right) \right].
\end{aligned}
$$

  - Productivity and residential externalities are substantial:

$$\lambda = 0.07, \quad \eta = 0.15$$

  - Productivity and residential externalities are highly localized:

$$\delta = 0.36, \quad \rho = 0.76$$

References

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