Econ 26610 Problem Set 1

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1 A Monocentric City Revisited

Consider a linear city embedded in a larger economy.

- The city has a continuum of locations indexed by their distance $d \in [-D, D]$ to the city center $d = 0$. Assume that D is large.
- A profit-maximizing representative firm produces a traded good in the city center $d = 0$ with a constant returns to scale technology $Y = A(L)L^f$, where L denotes total population in the city and L^f denotes the number of workers hired by the representative firm at wage rate W. Assume that $A(0) = 0$, $A'(L) > 0$, and $A''(L) < 0$. The final good serves as the numeraire of the economy; i.e. price of the final good is 1 and other prices are measured in unit of the final good.
- An absentee landlord in location d owns one unit of land and maximizes utility by choosing to either (1) supply the land to workers as housing and receive rent $R(d)$ or (2) use the land for agricultural purposes and produce R_0 unit of traded good. The absentee landlord draws utility from the consumption of the traded good.
- Homogeneous utility-maximizing workers make two decisions:
  1. They freely choose whether to move to the city by comparing the utility of living in the city with that of living outside of city which leads to consumption of $\bar{C}$.
  2. If they choose to move to the city, they then freely choose a location d to reside. Each location d is different in terms of rent $R(d)$ and commuting costs $\kappa(d)$ in terms of the traded good such that $\kappa(0) = 0$, $\kappa(d) = \kappa(-d)$, and $\kappa'(d) > 0 \; \forall d > 0$.

  The total number of workers in the economy (i.e. in and outside of the city) is large but nonetheless finite. Each worker supplies one unit of labor services at wage rate W, rents h units of land for housing at rental rate $R(d)$, and pays commuting costs

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$\kappa(d)$. Workers draw utility from the consumption of the traded good and those living in location d consume $C(d)$.

- The product, labor, and land markets are competitive. In other words, firms, workers, and landlords are price takers.

On Firm's Problem

1. What is the representative firm's optimization problem? Why is the agglomeration effect $A(L)$ conceptually a form of production externality?

2. Under what condition would the firm demand positive and finite L^f?
[Hint: Consider the implication of the firm's optimization problem.]

3. What is the relationship of L and L^f in equilibrium and why? What is city wage W as an expression of L in equilibrium? *[Hint: Think about total supply and demand of labor in the city.]*

On Worker's Problem

4. What is the consumption $C(d)$ of workers living in location d implied by their budget constraint? In equilibrium, do you expect $C(d)$ to depend on d? Why or why not? What is the implication for $R(d)$ and $\kappa(d)$?

5. Based on your answer in the previous question, is $R(d)$ increasing or decreasing in d for $d > 0$? What is the value of $R(d)$ charged by the marginal landlord at $d \in \{\bar{d}, -\bar{d}\}$ who is indifferent between renting out the land to workers and using it for agriculture?

6. What is the equation that characterizes the relationship between $\bar{d}$ and L? Why? Is the underlying assumption(s) leading to this result realistic? *[Hint: Think about supply and demand of land and recall that there is land on both sides of the city center $d = 0$.]*

7. What is the expression of $R(d)$ in terms of L, h, R_0, and $\kappa(d)$?
[Hint: Compare an arbitrary location d with a particular location hinted above.]

Equilibrium and Comparative Statistics

8. What is the equation that characterizes city population in equilibrium?
[Hint: You want to write an equation with L being the only unknown.]

9. Explain how you can use the equation from the previous question to derive the two figures on Week 1 Slides 36 and 37. Assume that $\kappa(d)$ is linear in d for this question.

10. How does L change if the outside value of land R_0 increases? Are workers in the city worse off? What is the economic intuition?

11. Assume that $A(L) = AL^{\alpha}$ for $\alpha < 1$ and show that the city would be larger if the firm were to internalize the production externality referred to in Q1.1.

Full Equilibrium

12. Assume that the whole economy consists of two identical cities $i \in \{1, 2\}$ (i.e. two cities share the same economic fundamentals $D, R_0, A(\cdot), \kappa(\cdot)$, but not necessarily the same equilibrium outcomes). Assume that total population in the economy is given by $\bar{L}$. What is the consumption of workers in the symmetric equilibrium?

13. How does equilibrium population in the two cities change if the outside value of land R_0 in both city increases to R_1? How is your answer different from Q1.10? Why?

2 A Tale of Two Cities

In this problem, we will try to solve for the equilibrium of a simple economy numerically and you may use any programming language that you prefer. The following packages/commands might be useful in this exercise:

- Julia: `NLsolve`.
- Matlab: `fsolve`.
- Python: `scipy.optimize.fsolve`.
- R: `optim`.

Consider an economy that consists of two cities. We use the same notations as in the previous problem and use subscript $i \in \{1, 2\}$ to denote corresponding values for each city. Moreover, we make the following functional/parametric assumptions:

- Let total population $\bar{L}$ be 10.
- Let $D = 10$ and $h = 1$ so that the the potential supply of housing is larger relative to the demand of housing.
- Let $A_i(\overline{L}_i)$ be given by

$$A_i(\overline{L}_i) = A_i \overline{L}_i^{\alpha_i}.$$

  This functional form implies that the two cities may be different in productivity even with the same population due to some fundamental difference in productivity or the strength of agglomeration effects.

- Let $\kappa_i(d)$ be given by

$$\kappa_i(d) = \kappa_i |d|^{\gamma_i}.$$

1. In this two-city economy, is $\overline{L}_1 = 10$ and $\overline{L}_2 = 0$ (or $\overline{L}_1 = 0$ and $\overline{L}_2 = 10$) always an equilibrium? Why or why not?
2. What is the one equation that characterizes the equilibrium of this two-city economy if both cities are active? *[Hint: The only unknown in this equation should be $\overline{L}_1$ or $\overline{L}_2$.]*
3. For the following two sets of parameters,
   (a) Plot the consumption of workers in each city as a function of $\overline{L}_1$ in one figure. Refer to these functions as $C_i^*(\overline{L}_1)$.

(b) Find equilibrium values of $\{\overline{L}_i, C_i^*\}$ such that both cities are active.

Case 1

Parameters	City 1	City 2
A_i	2.0	2.0
α_i	0.3	0.3
κ_i	1.0	1.0
γ_i	1.0	1.0
R_{0i}	0.15	0.10

Case 2

Parameters	City 1	City 2
A_i	2.0	2.0
α_i	0.5	0.5
κ_i	1.0	1.0
γ_i	1.0	1.0
R_{0i}	0.15	0.10

4. Given our parametric assumptions, in which case is the agglomeration force stronger when cities are not too small? Please justify your answer.

5. How and why is the shape of $C_1^*(\overline{L}_1)$ different in the two cases? Please explain briefly.

6. Are two-city equilibria in the two cases stable (i.e. if we deviate slightly from the equilibrium point, are we able to return to the same equilibrium)? Please justify your answer.

7. Is the two-city equilibrium in Case 1 efficient? In particular, is it possible for a Social Planner to choose a different $\overline{L}_1$ and a lump sum transfer from workers in one city to those in the other so that all workers are better off? What is the source of inefficiency?

8. What policy may help implement the welfare-improving allocation that you find above through market mechanism? Please describe briefly.

3 Short Answers

Answer the following questions with less than 5 bullet points each:

1. Suppose you have access to firm-level data for a large sample of firms in NYC and Newark for one industry. Suppose you observe that average costs are lower in the NYC sample compared to the Newark sample. Can you conclude from that evidence that NYC provides higher agglomeration externalities than Newark? Why or why not?

2. Explain why agglomeration externalities may explain why we observe firm concentrations outside the central business district.