EC 232 TA Session 5
Formulating Bellman

Feng Lin

11/01/2021

Comments for Midterm

- Many people are assuming away the non-interior case too quickly. For instance, if you realized Q2 is a Ben-Porath model, you should be extremely careful when you try to assume it away because that is an iconic feature of the model.
  - Related to this, Inada does not work in all cases. You should think a bit that it actually works before putting down Inada.
- Solving an ODE did not come to many people's mind when doing Q2d. There is an argument with which you do not have to solve the ODE explicitly (see Ben-Porath slide; it's there), which I would also give full credit this time. However, I think you do need to solve the ODEs to get the correct answers for later questions (so the expectation is that you can).

Comments for Midterm (Cont.)

- People may want to practice taking derivatives a little more (especially the chain rule). If you get the FOCs wrong, it is very likely that you will run into problems in later questions, since we usually ask you for the FOCs very early on.
- People should take a look another look at the guess-and-verify approach. Usually if we solve by guess-and-verify, we want to state clearly what our guess is and what conditions what we want to verify, and then go through them one-by-one. Many people might not realize Q2f is guess-and-verify or might not be familiar with the approach, so the response is a bit confusing.

A Formulation Problem

K. holds dollars and wants to buy a product that is only sold for jlutys. Jlutys are sold only on the black market, so K. must first search for a currency dealer to buy jlutys in exchange for dollars. She must then search for a vendor to purchase the object.

The exchange rate x (dollars per jlutys) offered by any black market currency dealer is drawn from a fixed distribution with cdf $Q(x)$, which has a continuous density $q(x)$. Any dealer is willing to exchange any amount K. wants.

The price p_j (in jlutys) demanded by any seller of the object is drawn from a fixed distribution with cdf $\Pi(\cdot)$, which has a continuous density $\pi(\cdot)$. If K. doesn't have sufficient jlutys, she must decline the offer and keep searching.

Each search has cost c (in dollars) and the discount rate is $r = 0$. Jlutys cannot be converted back into dollars, so K. will simply discard any unspent jlutys after she has purchased the object. K's objective is to minimize the expected discounted cost of purchasing the object.

Timing of Events

A Formulation Problem (Cont.)

(a) Let $J(m_j)$ denote the expected additional cost to K. if she has acquired m_j jlutys, is searching for a seller of the object, and has not yet paid the cost for the current search. Write the Bellman Equation for J.

A Formulation Problem (Cont.)

(b) Let $\Phi(x)$ denote the expected additional cost to K. if she has paid the search cost and has met a currency dealer who offers the exchange rate x. Write the Bellman Equation for Φ.

A Formulation Problem (Cont.)

(c) Let C^0 denote K.'s expected cost of purchasing the object, before she has begun. Write the Bellman equation for C^0.

Recursive Equilibrium

Ljungqvist and Sargent (2018)

Let

- x be a vector of state variables under the control of a representative agent;
- X be the vector of those same variables chosen by "the market";
- Z be a vector of other state variables chosen by "nature".

The representative agent's problem is characterized by the Bellman equation

$$v(x, X, Z) = \max_u \{R(x, X, Z, u) + \beta v(x_+, X_+, Z_+)\} \tag{0.1}$$

where $+$ denotes next period's value, and where the maximization is subject to the restrictions

$$x_+ = g(x, X, Z, u) \tag{0.2}$$
$$X_+ = G(X, Z) \tag{0.3}$$
$$Z_+ = \zeta(Z). \tag{0.4}$$

Here

- g describes the impact of the representative agent's controls u on his state x';
- G and ζ describe his beliefs about the evolution of the aggregate state.

The solution of the representative agent's problem is a decision rule

$$u = h(x, X, Z). \tag{0.5}$$

To make the representative agent representative, we impose

$$X = x,$$

but only "after" we have solved the agent's decision problem. Substituting equation 0.5 and $X = x$ into equation 0.2 gives the **actual** law of motion

$$X_+ = G_A(X, Z) \equiv g(X, X, Z, h(X, X, Z)). \tag{0.6}$$

A recursive competitive equilibrium is a policy function h, and actual aggregate law of motion G_A, and a perceived aggregate law G such that

1. Given G, h solves the representative agent's optimization problem;
2. h implies that $G_A = G$.